Real Analysis Part II
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Continuous Function

Let f be a function defined on an interval $[a, b]$. We shall now consider the behaviour of f at points of $[a, b]$

Continuity of a point

Def 1

Continuity at an interval point - A fun. f is said to be continuous at a point c, $a < c < b$ if

$$\lim_{x \to c} f(x) = f(c)$$

In other words, the function is continuous at c, if for each $\epsilon > 0$, $\exists$ $\delta > 0$ such that

$$|f(x) - f(c)| < \epsilon \text{ when } |x - c| < \delta$$

Def 2 A function f is said to be continuous from left at c if

$$\lim_{x \to c-0} f(x) = f(c)$$

Also f is continuous from the right at c if

$$\lim_{x \to c+0} f(x) = f(c)$$

Clearly a function is continuous at c if and only if it is continuous from left as well as from the right.

Def 3 Continuous at an end point

A function f defined on a closed interval $[a, b]$ is said to be continuous at the end point a if it is continuous from the right at a ie

$$\lim_{x \to a+0} f(x) = f(a)$$

Also the function is continuous at the end point b of $[a, b]$ if

$$\lim_{x \to b-0} f(x) = f(b)$$

Thus a function f is continuous at a point c if (i.) $\lim_{x \to c} f(x)$ exists, and

ii.) limit equals the value of the function at $x = c$

Continuity in an interval

A function f is said to be continuous in an interval [a,b] if it is continuous at every point of the interval.

Discontinuous function

A function is said to be discontinuous at a point c of its domain if it is not ~~not~~ continuous there at the point c is then called a point of discontinuity of the function.